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Sprites as evidence of vertical gravity wave structures above mesoscale thunderstorms

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Abstract. Large area multicell thunderstorms lead to the formation of vertically oriented cylindrical structures of gravity waves at mesospheric altitudes closely resembling those observed in optical emissions associated with transient luminous glows called sprites.

Introduction

Sprites are vertically oriented clusters of luminous glows lasting several milliseconds at ~50 to 90 km altitudes above large mesoscale thunderstorms [e.g., *Sentman et al.,* 1995; *Rairden and Mende,* 1995, and references therein] that are believed to be produced by quasi-electrostatic thundercloud fields [e.g., *Pasko et al.,* 1997a, and references therein]. They typically appear several hours after the onset of a storm, even in extremely active (high flash rate) storms [*Lyons,* 1996]. We propose that the occurrence of sprites may be facilitated by vertical gravity wave (GW) structures supported by mesoscale storm systems. Gigantic multicell storms with nearly circular cloud shape covering an area more than 300×300 km^2, called Mesoscale Convective Complexes (MCC) [e.g., *Djuric,* 1994, p. 220], are expected to significantly enhance the energy launched upward in the form of GW. These types of storms are often observed in the Great Plains of North America, which is a region in which sprites are commonly observed [e.g., *Sentman et al.,* 1995; *Rairden and Mende,* 1995; *Lyons,* 1996]. The oscillations of air due to penetrative convection at thundercloud tops can lead to the upward launching of GW with periods near the local Brunt-Väisälä period at the source altitude and with the velocity of vertical oscillations as high as ~30 m/s extended over an area ~10 km by 10 km [*Pierce and Coroniti,* 1966; *Stull,* 1976]. GW arise mainly from mechanical forcing by vertical displacements associated with oscillatory updrafts and down drafts rather than thermal effects [*Fowell et al.,* 1992]. *Rowland et al.* [1996] suggested that modulation of atmospheric neutral density associated with GW motions may cause spatial variations in high altitude (>80 km) optical emissions and ionization produced by lightning electromagnetic pulses. Vertically oriented structures observed in sprites at ~50-90 km altitudes can also be expected to be related to GWs; however, until now no particular mechanism of GW generation leading to vertically striated structures has been identified.

In this paper we use a cylindrically symmetric treatment of the upward propagation and diffraction of GW in an isothermal compressible atmosphere to demonstrate that GW launched by large area MCC can lead to significant (tens of %) modulation of atmospheric density at mesospheric altitudes in vertically oriented cylindrical structures closely resembling those observed in optical emissions associated with sprites.

Model

The generation of GW by updrafts and downdrafts associated with penetrative convection [e.g., *Stull,* 1976] is considered as a linear boundary value problem in cylindrical coordinates (r, z, φ), where z is altitude. The vertical velocity v_{zo} is a given function of r, φ and time t at the thundercloud top (z_o=10 km). We use linearized equations of motion, of adiabatic state, and of continuous mass conservation [e.g., *Hines,* 1960], and assume variation of $\sim\exp[i(\omega t - k_z z)]$, where ω is the wave frequency and k_z is the complex wave number in the z direction. Hankel transformation [e.g., *Bracewell,* 1986, p. 244; *Poularikas,* 1995, p. 721] leads to the following solution for vertical velocity v_z:

$$v_z = v_o\, e^{\frac{\omega_a}{c_s}z'} \int_0^\infty dk_r\, k_r\, A_m(k_r)\, J_m(k_r r)\, e^{i(\omega t - k_z z' - m\varphi)} \tag{1}$$

where $z' = z - z_o$, v_o is the amplitude of vertical wave velocity at $z' = 0$, k_r is the radial wave number, m is the integer azimuthal number, the radial spectral function $A_m(k_r) = \mathcal{H}_m\{v_{zo}(r)\}/v_o$, where $\mathcal{H}_m\{\cdot\}$ is the Hankel transform operator of the m-th order, $\omega_a = g\gamma/(2c_s)$ is acoustic cut-off frequency, c_s is speed of sound, γ is the specific heats ratio, and g is the gravitational constant. The dispersion relation can be shown to be

$$k_z^2 = \frac{\omega_g^2 - \omega^2}{\omega^2/k_r^2} + \frac{\omega^2 - \omega_a^2}{c_s^2} \tag{2}$$

where $\omega_g = g\sqrt{\gamma - 1}/c_s$ is the Brunt-Väisälä frequency. The fractional density perturbation $\tilde{\rho}/\rho$ associated with the wave motion is similar in form to (1) with the initial (in general complex) amplitude $\tilde{\rho}_o/\rho$ at the thundercloud top defined by v_o as:

$$\frac{\tilde{\rho}_o}{\rho} = -\frac{g(\gamma-1)k_z + i(\omega^2 - \gamma\omega_g^2/2)}{g - c_s\omega_a - ik_z c_s^2}\,\frac{v_o}{\omega} \tag{3}$$

For a source perturbation in the form $v_{zo} = v_o\, J_m(k_{ro}r)$ $\exp[i(\omega t - m\varphi)]$ for $r \leq r_i$ (v_{z0}=0, for $r > r_i$) with characteristic radial wavelength λ_{ro}=$2\pi/k_{ro}$ and effective aperture radius r_i such that $J_m(k_{ro}r_i)$=0, $A_m(k_r)$ can be calculated using Lommels integrals [e.g., *Korn and Korn,* 1961, p. 731] and expressed in the form:

$$A_m(k_r) = \begin{cases} -\frac{k_{ro}r_i}{k_r^2 - k_{ro}^2}\, J_m(k_r r_i)\, J_{m+1}(k_{ro}r_i), & k_r \neq k_{ro} \\ \frac{r_i^2}{2}\, [J_{m+1}(k_{ro}r_i)]^2, & k_r = k_{ro} \end{cases} \tag{4}$$

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Sprites are believed to be produced as a result of the electrical breakdown of the neutral atmosphere by large mesospheric electric field transients following intense cloud-to-ground discharges [e.g., *Pasko et al.,* 1997a, and references therein]. Since the ionization and the optical emission excitation rates are sensitive functions of atmospheric neutral density and may be enhanced several orders of magnitude in response to only several percent depletions in $\tilde{\rho}/\rho$ [*Rowland et al.,* 1996; *Pasko et al.,* 1997a], the appearance and spatial structure of sprites may be related to GW formations.

Results

Since the dispersion relation $k_z = k_z(\omega, k_r)$ (2) has the same form as in cartesian coordinates, the previously derived dispersion properties of GW [e.g., *Hines,* 1960; *Gossard and Hooke,* 1975] are applicable to the cylindrical case. Assuming $k_z > \omega_a/c_s$ (typically valid for λ_z <100 km) and considering solutions in gravity wave range of frequencies ($\omega \leq \omega_g$), known solutions for waves with upward propagating energy (angle θ between the group velocity $\vec{U}$ and the vertical is defined as $\theta = \arccos(\omega/\omega_g)$) and downward propagating phase ($\vec{U} \perp \vec{V}$, where $\vec{V}$ is phase velocity) can be utilized [e.g., *Hines,* 1960]. The group velocity components are

$$U_r = \frac{\lambda_r \omega}{2\pi}\left(1 - \frac{\omega^2}{\omega_g^2}\right) \qquad (5)$$

$$U_z = \frac{\lambda_r \omega^3}{2\pi \omega_g^2}\sqrt{\frac{\omega_g^2}{\omega^2} - 1} \qquad (6)$$

For a given λ_r, U_z is maximum at $\omega/\omega_g=\sqrt{2/3}$ and U_r at $\omega/\omega_g=1/\sqrt{3}$, and $U_r = U_z$ at $\omega/\omega_g=1/\sqrt{2}$. In the range $\sqrt{2/3} < \omega/\omega_g < 1$, both U_r and U_z are rapidly decreasing functions of frequency. In order to produce measurable effects associated with GW, cloud to ground discharges producing sprites [*Pasko et al.,* 1997] should remove charge roughly (± 50 km) from the same area of MCC as occupied by penetrative convection. Waves generated with frequencies $\omega < \omega_g/\sqrt{2}$ ($U_r > U_z$) travel predominantly in horizontal direction out of the storm and do not reach mesospheric altitudes above MCC where sprites are observed. Assuming $U_z > U_r$ the minimum U_z required for GW to reach $\sim$ 90 km on the time scale of a typical storm ($\sim$1 hour) is $U_z \sim 22$ m/s. For the frequency range of interest ($0.7 < \omega/\omega_g < 0.95$) the minimum λ_r of the waves should be in the range 17-25 km, implying that only large MCCs can produce mesospheric density modifications that may facilitate the production of sprites. The observed several hours delay between the onset of a thunderstorm and the appearance of sprites [*Lyons,* 1996] may thus be a consequence of the time required for the GW to reach mesospheric altitudes.

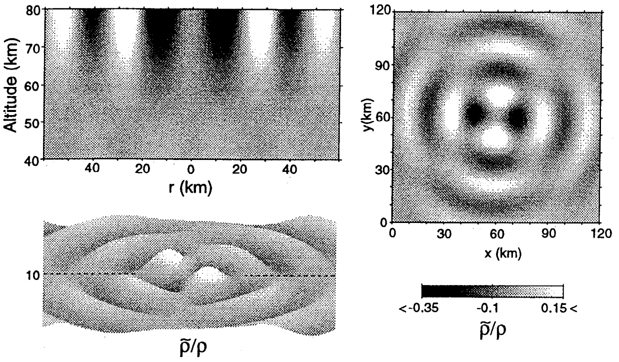

Figure 1. The fractional density change $\tilde{\rho}/\rho$ associated with a single mode GW. *Left panel:* A cross sectional vertical (r, z) view in φ=0 plane, where z is marked as altidude. The bottom image illustrates three dimensional distribution (shown not to scale) of $\tilde{\rho}/\rho$ at altitude 10 km. *Right panel:* A cross sectional horizontal (x, y) view in z =70 km plane. Parameters used: γ=1.4, g=9.8 m/s^2, c_s =300 m/s, ω_g=0.0207 rad/s, ω_a=0.0229 rad/s, (scale of atmosphere $H = c_s^2/g\gamma \simeq 6.7$ km), λ_{ro}=25 km, m=2, ω/ω_g=0.95, v_o=2.5 m/s.

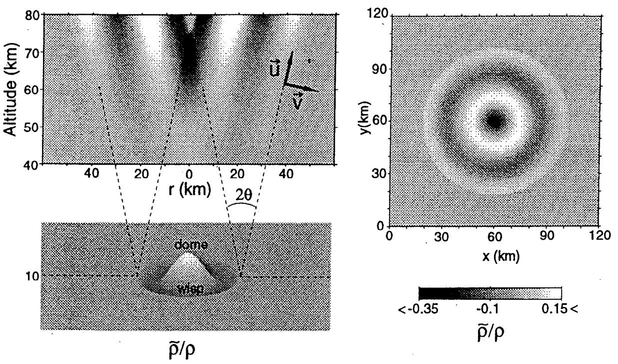

Figure 2. The fractional density change $\tilde{\rho}/\rho$ associated with the diffraction of cylindrical GW on a circular aperture with radius r_i=22 km. The format and parameters used are the same as indicated in Figure 1, except m=0.

Single radial mode solutions can be obtained from (1) by assuming $r_i \rightarrow \infty$, since $A_m(k_r) \rightarrow 1/k_{ro}\delta(k_r - k_{ro})$. Figure 1 shows results of calculations of $\tilde{\rho}/\rho$ for m=2, λ_{ro}=25 km, $\omega = 0.95\omega_g$, and v_o=2.5 m/s (corresponding to $\tilde{\rho}/\rho \simeq 2.5\times10^{-3}$ at $z = z_o$). Results indicate that even for this simplest single mode case the system naturally supports vertically oriented columnar structures. These single mode structures are simply vertical for m=0. For $m \neq 0$, the structures are in the form of a helix, following lines $\varphi(z) = k_z z/m$, as function of altitude.

Figure 2 shows results of calculations for an azimuthally symmetric (m=0) penetrative element consisting of characteristic "wisp" and "dome" regions [e.g., *Stull,* 1976]. To find the solution in this case we define $r_i = 5.5201/k_{ro}$, corresponding to the second zero of $J_o(k_{ro}r_i)$ to determine the spectral function $A_o(k_r)$ (4), and then integrate (1) numerically. The problem under study here is equivalent to the diffraction of a single mode cylindrical GW incident from below on a circular aperture with radius r_i. Each point on the aperture is a source of secondary Huygens'wavelets [e.g., *Elmore and Heald,* 1969, p. 323]. At each observation point contributions of all the Huygens' wavelets emanating from different points on the aperture are superposed. Dashed lines in Figure 2 schematically show directions of waves emitted from two radially opposite points on the aperture, defining spatial boundaries for allowed wave motions. Directions of group ($\vec{U}$) and phase ($\vec{V}$) velocities are also shown for reference ($\vec{U}$ being directed along the lines of constant phase, i.e., $\vec{U}\perp\vec{V}$). The upward propagation of energy is possible only for waves with downward propagation of phase. The characteristic focusing point is at an altitude just below $z_f' = r_i/\sqrt{\omega_g^2/\omega^2 - 1} \sim 67$ km, (below

$z = z'_f + z_o$=77 km) with a relatively large localized negative density perturbation ($\tilde{\rho}/\rho \simeq$-0.3) in a region less than 10 km in lateral extent.

The modulation of the neutral atmosphere is likely to lead to a significant modification of the structure of optical emissions associated with sprites [*Pasko et al.*, 1997a]. Several tens of % depletions in the neutral density would lower the effective altitude of the breakdown region by only several km. However, the characteristic scale of atmospheric conductivity at 70-90 km altitudes is mostly determined by lower ionospheric electrons and is typically expected to be 1-2 km or even less [e.g., *Pasko et al.*, 1997a]. Since the penetration of postdischarge electric fields to higher altitudes is dependent upon the effective dielectric relaxation time in the conducting medium at these altitudes (see [*Pasko et al.*, 1997a]), several km changes in the altitude of breakdown ionization can be instrumental in initiating the breakdown process associated with sprites. Additional factors showing potential importance of GW created modulation of the neutral density at mesospheric altitudes are related to possibility of initiation of small scale (lateral extents <100 m) streamer type processes with very fast (~10 km/ms) upward/downward propagation [*Pasko et al.*, 1997b] which under certain circumstances may significantly modify the breakdown region associated with sprites consistent with recent observations of highly stratified filamentary structure of optical emissions [e.g., *Sentman et al.*, 1996; *Taylor and Clark* 1996; *Stanley et al.*, 1996; *Fukunishi et al.*, 1996].

Discussion

Although based on physically reasonable assumptions, the proposed mechanism still remains a subject for future experimental verification, mainly due to the limited amount of available experimental data related to mesospheric GW generated by thunderstorms, especially atmospheric neutral density variations associated with them. The currently available data on short period (~5-8 min) GW generated by thunderstorms [*Balachandran*, 1980; *Larsen et al.*, 1982; *Taylor and Hapgood*, 1988], and the presence in the mesosphere of density fluctuations up to several tens of percent with characteristic horizontal wavelengths ~10 km [*Hines*, 1960; *Fritts et al.*, 1989; *Walterscheid and Schubert*, 1990], generally support the mechanism put forth in this paper.

GWs break at high altitudes due to nonlinear overturning which occurs when the wave velocity exceeds its phase velocity, which itself may lead to a complicated spatial structure of wave motions with significant localized enhancements of the density perturbations [*Walterscheid and Schubert*, 1990]. Analysis of results presented in Figure 1 indicates that at ≃80 km altitude the horizontal wave velocity is still two times less than its phase velocity (source amplitude v_o=2.5 m/s was in part chosen to justify a linear analysis in the altitude region of interest, up to ~80 km). A nonlinear analysis is required in order to accurately follow wave dynamics above this altitude or for higher values of v_o.

Our simplified single frequency analysis assumes steady-state emission of GWs by the penetrative element with finite radial extent. The fact that such oscillations would actually be limited in time [e.g., *Stull*, 1976] would lead to the excitation of a range of frequencies [*Fowell et al.*, 1992; *Alexander*, 1996]. Of the total energy emitted in (ω, k_r) space, only the fraction corresponding to frequencies comparable to the ambient Brunt-Väisälä frequency ($\omega \sim 0.7$-$0.95\omega_g$) and horizontal wavelengths ~20-25 km would propagate upward over the time scale of the thunderstorm and would lead to vertical density structures producing sprites during intense lightning discharges. Recent squal line simulations [*Alexander*, 1996] show that up to 35% of GW energy may correspond to modes with period 8 min ($\omega \sim$0.013 rad/s), horizontal wavelengths 12 and 24 km, and vertical velocity amplitudes 1.8 m/s at z=13 km.

Waves with $\lambda_r \ll$20-25 km ($\omega/\omega_g \sim$0.7-0.95) are slow ($U_z \propto \lambda_r$), and can be effectively removed by eddy viscous damping [*Fritts*, 1984]. Using a maximum value of eddy kinematic viscosity $\nu_{\rm eddy} \sim 10^7$ cm^2/s for lower ionospheric altitudes [*Gossard and Hooke*, 1975, p. 222] we estimate the minimum allowable wavelength as $\lambda_{\min} \sim 2\pi\sqrt{\nu_{\rm eddy}/\omega} \sim 1.4$ km (a different approach to this problem leads to a factor of 2π higher values, thus giving $\lambda_{\min} \sim$9 km [*Hines*, 1960]). GWs can heat the neutral atmosphere through loss of wave energy due to viscous friction as $dT/dt \simeq \nu_{\rm eddy} v_z^2 k_z^2/c_{\rm v}$ [*Hines*, 1965; *Taylor*, 1979], where T is temperature in °K and $c_{\rm v}$ is the specific heat at constant volume. For the parameters shown in Figure 1 (max(v_z) ≃70 m/s, $\lambda_z \simeq$75 km) we estimate $dT/dt \simeq$ 0.35 °K/hour which on a time scale of ~10 hours may compete with integral daily heating produced by solar radiation (see discussion in [*Hines*, 1965]). This type of mesospheric heating associated with thunderstorms is confirmed experimentally [*Taylor*, 1979].

Waves with $\lambda_r \gg$20-25 km ($\omega/\omega_g \sim$0.7-0.95) have $U_r > U_z$ and travel out of the storm along slanted paths. A part of the energy of the total wave spectrum $A_m(k_r)$ corresponding to large wavelengths ($\lambda_r > \lambda_{\max}$, where $\lambda_{\max} = 2\pi c_s/\omega\sqrt{(\omega_g^2/\omega^2 - 1)/(\omega_a^2/\omega^2 - 1)}$) corresponds to evanescent waves and does not propagate upward.

In a more realistic non isothermal atmosphere, ω_g increases with altitude from ~0.01 rad/s to ~0.02 rad/s in a narrow region around 10 km, remains approximately constant up to ~45 km, decreases relatively slowly to a value ~0.015 rad/s at ~65 km altitude and then increases again to ~0.02 value at ~80 km [e.g., *Gossard and Hooke*, 1975, p. 19]. Only wave groups generated at 10 km altitude with $\omega \leq$ 0.015 rad/s would have a chance to reach sprite producing mesospheric regions. These waves initially would propagate with large angles ~45° with the vertical and have characteristic wavelength ratio $\lambda_z/\lambda_r = 1/\sqrt{\omega_g^2/\omega^2 - 1} \sim 1$, becoming more and more vertically directed ($\theta \rightarrow 0, \lambda_z/\lambda_r <1$) as they approach the altitude of minimum ω_g at ~ 65 km. Waves with higher frequencies would be removed by reflection, while waves with much lower frequencies would propagate horizontally out of the storm. Reflection at altitudes ~65 km may form standing wave structures, doubling the amplitude of the density perturbation.

The cylindrical model does not allow the investigation of effects of ambient atmospheric winds on GW motions (i.e., critical levels [e.g., *Brethenton*, 1966]) and is accurate only for quasi-static storms (similar to convective storm cells associated with sprites observed in northern Peru [*Sentman et al.*, 1995b]), or storms moving with ground related wind speed (called S(0) case in [*Fowell et al.*, 1992]).

Diffraction focusing points at mesospheric altitudes (e.g., z'_f ~60 km) with large density perturbations would generally occur for large aperture sizes r_i (for ω/ω_g in the range 0.7-0.95, r_i should be ~20-60 km for isothermal atmosphere, and even greater for a more realistic atmosphere) and can thus be most readily excited by multicell storms with large horizontal extent (e.g., MCC). For small aperture sizes r_i, the focusing point z'_f moves down to the thundercloud top and GW as shown in Figure 2 take the form of two cylindrical "rays" emerging upward with angle 2θ between them, each having width $\sim r_i$, and producing no perturbations at mesospheric altitudes right above the aperture. Rays take the form of cylindrical filaments for modes with $m \geq 1$. If a range of wave frequencies is excited, rays with different angles θ appear as originating from a single point at the thundercloud top (ignoring scale r_i). These types of structures were demonstrated in numerical simulations of convectively generated GW [*Fowell et al.*, 1992; *Alexander*, 1996] and are sometimes observed in sprite phenomena [*Winckler et al.*, 1996; *Stanley et al.*, 1996].

Summary

Cylindrical GWs generated by penetrative convection in large area MCCs with periods comparable to the local Brunt-Väisälä period (~5-8 min) and horizontal wavelengths ~20-25 km can form mesospheric cylindrical structures closely resembling those observed in optical emissions associated with sprites. Diffraction effects of GW produced by large area sources can lead to focusing of the wave energy in localized regions of space ($\leq$10 km in lateral extent), with significant associated enhancements of atmospheric perturbations.

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